Enrollment No: _____________________ **Exam Seat No:** _____________________

C.U.SHAH UNIVERSITY

Summer Examination-2017

Subject Name: Banach Algebras

Subject Code: 5SC04BAE1 **Branch: M.Sc. (Mathematics)**

Semester: 4 **Date: 18/04/2017** **Time: 10:30 To 01:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

a. Define: Topological zero divisor. **(01)**
b. Define: Radical of an algebra. **(01)**
c. Define: Semi simple algebra. **(01)**
d. If I is a proper left ideal in algebra A then prove that $\bar{I}$ is a left ideal in A. **(02)**
e. Define: Weak* topology. **(02)**

Q-2 **Attempt all questions** **(14)**

a) Let $C^{'}[a,b] = \{f \in C[a,b] \mid f^{'}$ exists & $it\ is\ continuous\ on\ [a,b]\}$ with pointwise operations. Show that $C^{'}[a,b]$ is commutative Banach algebra with identity where $\|f\|_1 = \|f\|_\infty + \|f^{'}\|_\infty$. **(07)**

b) Let A be Banach algebra with identity and G be the set of all regular in element in A. Let $x \in A$ with $\|e - x\| < 1$ then prove that **(07)**
(i) $x \in G$ and $x^{-1} = \sum_{n=0}^{\infty}(e-x)^n$
(ii) G is open in A.

OR

Q-2 **Attempt all questions** **(14)**

a) Let A be algebra without identity and $A_e = \{(x,a) \mid x \in A, a \in \mathbb{C}\}$. **(07)**
Define $(x,a) + (y,b) = (x+y, a+b)$
$\lambda\,(x,a) = (\lambda x, \lambda a)$
$(x,a)\,(y,b) = (xy + ay + bx, ab)$
$\|(x,a)\| = \|x\| + |a|$
Then prove that A_e is an algebra with identity. Also if A is complete then prove that A_e is complete.

b) Let A be a Banach algebra with identity and $x \in A$. Then prove that $r(x) = \lim_{n\to\infty}\|x^n\|^{1/n}$. **(07)**

Q-3 **Attempt all questions** **(14)**

a) Let A be a Banach algebra with identity. In usual notation prove that **(07)**
(i) $Z \subset S$.
(ii) $bd\ S \subset Z$.

b) Let A be a Banach algebra with identity over $\mathbb{C}$. Then prove that $A \cong \mathbb{C}$ in each of the following cases **(07)**
(i) Zero is the only topological divisor for A.
(ii) there exists $M > 0$ such that $\|xy\| \geq M\,\|x\|\,\|y\|\ \forall\ x, y \in A$.

OR

Q-3 **Attempt all questions** **(14)**

a) Let A be a Banach algebra with identity then prove that $\sigma(x) \neq \emptyset$. **(06)**

b) State and prove Gal'fand – Mazur theorem. **(04)**

c) Let B be Banach sub algebra of a Banach algebra A and $x \in B$ then prove that $bd\ \sigma_B(x) \subset bd\ \sigma_A(x)$. **(04)**

SECTION – II

Q-4 **Attempt the Following questions** **(07)**

a. Define: Gelfand topology. **(02)**
b. Define: Involution **(02)**
c. Define: Banach * algebra. **(01)**
d. Define: C* algebra. **(01)**
e. Every C* algebra is B* algebra. True or False. **(01)**

Q-5 **Attempt all questions** **(14)**

a) Prove that $A \mid R$ is semi simple Banach algebra if A is semi simple Banach algebra and R is closed two sided ideal in A. **(07)**

b) Let A be Banach algebra with identity. Prove that every ideal contained in a maximal ideal in A. **(07)**

OR

Q-5 **Attempt all questions** **(14)**

a) Let A be Banach algebra with identity. Prove that **(07)**
$Rad\ (A) = \{r \in A \mid e - xr \text{ is regular } \forall\ x \in A\}$.

b) Prove that there is one to one correspondence between the set of all non zero complex homomorphism $\Delta(A)$ and the collection of all maximal ideal in A where A is commutative Banach algebra. **(07)**

Q-6 **Attempt all questions** **(14)**

a) State and prove Gelfand Naimark theorem. **(10)**

b) Let A be a B^* algebra and $x \in A$ be normal then prove that $\|x^2\| = \|x\|^2$. **(04)**

OR

Q-6 **Attempt all questions** **(14)**

a) State and prove Banach – Stone theorem. **(07)**

b) Let X be a compact T_2 space. Then prove that the maximal ideal space of $C(X)$ is homeomorphic to X. **(07)**